Ethnomathematics as a new research field, illustrated by studies of mathematical ideas in African history

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Preliminary remarks

The organisers of this conference on "New Trends in the History and Philosophy of Mathematics" asked me to "...view the history and philosophy of mathematics from the angle of studies in 'Ethnomathematics'", and "...hear about [my] methodological reflections, especially in case studies as examples, but not a review of general trends". As it is not easy to do justice to the proposed, partially opposite tasks, I should like to refer the reader, beforehand, to some of my earlier studies that review some general trends and/or present overviews.
For a historical and philosophical reflection on the role of human, cultural activity ('Tätigkeit' in German) in the development of early geometrical thinking, I should like to refer to my study *Zum erwachenden geometrischen Denken* (1985), published in German under the title *Ethnogeometrie* (1990) with a preface by Peter Damerow. An English language version under the title *Culture and the awakening* of geometrical *thinking* with a preface by Dirk Struik is currently in press (Gerdes, 2000). For an introduction to mathematical ideas in the history of Africa south of the Sahara, see Claudia Zaslavsky's classical *Africa Counts* (1973), and for an update and literature overview see my paper in *Historia Mathematica* (Gerdes. 1994). For an overview of the development of ethnomathematics in relationship to concerns in mathematics education, see my paper in the *International Handbook of Mathematics Education* (Gerdes, 1996). For a reflection on ethnomathematics and mathematics teacher education, see my paper in the international *journal of Mathematics Teacher Education* (Gerdes, 1998).
The organisers suggested as title for my talk "Ethnomathematics as a new research field illustrated by new discoveries in 'African Mathematics'". As can be seen, I changed the second part of the title a little bit "...illustrated by studies of mathematical ideas in African history". There are several complementary reasons to avoid in a meeting of historians and philosophers of mathematics the expression 'African Mathematics'. One reason is to avoid any connotation with expressions as 'German Mathematics', 'One reason is to avoid any connotation with expressions as 'German Mathematics', 'French Mathematics', ...and the political-ideological environment of nazism in which they were developed *(cf.* the contribution to this volume by Herbert Mehrtens). An other reason is not to be dragged into a debate like the one on African Philosophy (and ethnophilosophy) (cf. Houtondji,

1976,1997; Houtondji & Wiredu, 1984): What is philosophy?, Where ends wisdom and starts philosophy?,... I prefer to speak about mathematical activities and ideas, about mathematical aspects of cultural phenomena, about mathematical elements in cultural contexts (cf. Gerdes, 1997a: 402). With this remark, we have already arrived at the notion of ethnomathematics. In this paper I will discuss ethnomathematics and related concepts, present two examples of studies of mathematical ideas in African history, and conclude with a reflection about ethnomathematics and mathematical thinking.

Ethnomathematics / ethnomathematicology

Ethnomathematics (or ethnomathematicology) is relatively new as a field of research. It may be described as the study of mathematical ideas and activities as embedded in their cultural context. As views of mathematics as "culture-free" and "universal" have been rather dominant in the academia, ethnomathematics emerged relatively late.

Otto Raum with his *Arithmetic in Africa* (1938), Dirk Struik with his On the *sociology o f mathematics* (1942), Leslie White with his The *locus of mathematical reality: an anthropological footnote* (1947), and Raymond Wilder with his *The cultural basis o f mathematics* (1950) may be counted among the (relatively) isolated forerunners of ethnomathematicology.

The Brazilian Ubiratan D'Ambrosio launched in the 1970's his *ethnomathematical program (cf.* D'Ambrosio, 1985, 1997), as a methodology to track and analyse the processes of generation, transmission, diffusion and institutionalisation of mathematical knowledge in diverse cultural systems (D'Ambrosio, 1990: 78). He developed his ideas inspired by a reflection about major problems of mathematics education in the 'Third World'. In contrast to "academic mathematics", i.e. the mathematics which is taught and learned in schools and universities, D'Ambrosio called ethnomathematics "the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on" (D'Ambrosio, 1985a: 45). That so many children fail in mathematics is due to "the mechanism of schooling [that] replaces these practices by other equivalent practices which have acquired the status of mathematics, which have been expropriated in their original forms and returned in a codified version" (D'Ambrosio, 1985a: 47).

Emergence of several concepts

D'Abrosio's ethnomathematics was not the only concept that emerged in the context of reflections on mathematics education in the 'Trird World' and later found an echo in other parts of the globe. Colonial education had presented mathematics generally as something rather 'western', 'European', as an exclusive creation of 'white men'. The hasty curriculum transplantation —during the 1960's— from the highly industrialised nations to 'Third World' countries did not change fundamentally this image.

During the 1970's and 1980's, there emerged among teachers and mathematics educators in developing countries and later also in other countries a growing resistance to this presentation (cf. e.g. Njock, 1985; Powell & Frankenstein, 1997), against the racist and (neo) colonial prejudices, against the eurocentrism, that it reflects. It was stressed that beyond the 'imported school mathematics' there have existed also other forms of mathematics.

In this context, various concepts have been proposed to contrast with the *'academic* mathematics' / *'school* mathematics' *(i.e.,* the school mathematics of the transplanted, imported curriculum):

- *'Indigenous* mathematics' (cf. e.g. Gay & Cole, 1967; Lancy, 1978). Criticizing education of Kpelle children (Liberia) in 'western-oriented' schools —they "are taught things that have no point or meaning within their culture" (1967: 7) -Gay and Cole propose a creative mathematical education that uses the indigenous mathematics as starting point;
- Sociomathematics of Africa (Zaslavsky, 1973: 7): "the applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon the evolution of their mathematics";[1]
- *Informal* mathematics (Posner, 1978, 1982; Ascher & Ascher, 1981): mathematics that is transmitted and that one learns out-side the formal system of education;
- *Mathematics in the (African) socio-cultural environment* (S. Doumbia, S. Touré (Côte d'Ivoire), 1984): the mathematics of African games and craft work that belongs to the socio-cultural environment of the child should be integrated in the mathematics curriculum;
- *Spontaneous* mathematics (D'Ambrosio, 1982): each human being and each cultural group develops spontaneously certain mathematical methods;
- *Oral* mathematics (Carraher et *al.,* 1982, 1987; Kane, 1987): in all human societies there exists mathematical knowledge that is transmitted orally from one generation to the next;
- *Oppressed* mathematics (Gerdes, 1982,1985a): in class societies (e.g., in the countries of the Trird World' during the colonial occupation) there exist mathematical elements in the daily life of the populations, that are not *recognized* as mathematics by the dominant ideology;
- *Non-standard* mathematics (Carraher et *al.,* 1982; Gerdes, 1982, 1985á; Harris, 1987,1997): beyond the dominant standard forms of ‘academic' and 'school' mathematics, other mathematical forms have developed in the whole world and in each culture. Carraher (= Nunes) also uses the expression street mathematics for mathematical ideas developed in the 'streets', outside the school context *(cf.* Carraher, 1988);

[1] D'Ambrosio used in 1976 the same term in the context of Brazil. See U. D'Ambrosio (1976).

- Hidden or frozen mathematics (Gerdes, 1982, 1985, 1986,1990, 2000): although, probably, the majority of mathematical knowledge of the formerly colonized peoples has been lost, one may try to reconstruct or 'unfreeze" the mathematical thinking, that is 'hidden' or 'frozen' in old techniques, like, e.g., that of basket making
- *Folk* mathematics (Mellin-Olsen, 1986): the mathematics (although often not recognized as such) that develops in the working activity of each of the peoples may serve as a starting point in the teaching of mathematics;
- *People's* mathematics as a component of people's education in the context of the struggle against apartheid in South Africa (Julie, 1989);
- Mathematics *codified in know-how* (Ferreira, 1991);
- *Implicit* and *non-professional* mathematics (Ascher & Ascher, 1981; Zaslavsky, 1994).[2]

Various aspects illuminated by the aforementioned concepts have been gradually incorporated under the more general denominator of ethnomathematics. This process has been accelerated by the creation of the *International Study Croup on Ethnomathematics* [ISGEm] in 1985.

First level of definition

On the one hand, if ethnomathematics is considered as the *mathematics of a certain (sub)culture,* then 'academic mathematics' is also a concrete example of ethnomathematics. On the other hand, when all ethnomathematics is mathematics, why call it ethnomathematics? And not simply the mathematics of this and that (sub)culture? Doing so, ISGEm defines ethnomathematics also at another level, as a research field, that reflects the consciousness of the existence of many mathematics, particular in a way to certain (sub)cultures.

Second level of definition

As a research field, ethnomathematics (or ethnomathematicology like ethnomusicology) may be defined as the *cultural anthropology of mathematics and*

[2] At the end of their book Code *of the Quipu.* Ascher & Ascher present the following interesting comments on the development of mathematics and of mathematical activities: "Mathematics arises out of, and is directly concerned with, the domain of thought involving the concepts of number, spatial configuration, and logic. In Western culture a professional class, called mathematicians,..., deals solely and exclusively with these concepts. Examples of other groups involved in mathematical endeavors are accountants, architects, bookies, construction engineers, landscape designers, navigators, and system analysts. Non-professional mathematics, as practiced by these groups... may often be implicit rather than explicit. When these mathematical endeavors are implicit, they are none the less, mathematics. Because of the provincial view of the professional mathematicians, most definitions of mathematics exclude or minimize the implicit and informal. It is, however, in the nature of any professional class to seek to maintain its exclusivity and to do this, in part, by recreating the past in terms of unilinear progress towards its own present" (Ascher & Ascher, 1981: 158-159).

mathematical education, or in the formulation of D'Ambrosio in 1977: *"Ethnoscience* as the study of scientific and, by extension, technological phenomena in direct relation to their social, economic and cultural background" (D'Ambrosio, 1987: 74). In this sense it includes "the study of mathematical ideas of non literate peoples", which was the Ascher's definition of ethnomathematics in 1986.

Contrast with 'ethnographers' conceptualisations of ethnoscience

Among ethnographers and anthropologists the concept of 'ethnoscience' is used sience the end of the 19th century. Their use of the concept seems to be either more restricted or differently ideologically loaded than the concept as it is used nowadays by mathematicians and mathematics educators.
In the ethnological dictionary of Panoff and Perrin (1973) two definitions of the concept of ethnoscience are presented. In the first case, it is a "branch of ethnology that dedicates itself to the comparison between the positive knowledge of exotic societies and the knowledge that has been formalized in the established disciplines of western science" (Panoff & Perrin, 1973: 68). This definition raises immediately some questions, like: "What is *positive* Knowledge?"; "In what sense exotic?" and "Does there exist a western science?" In the second case, "each application of one of western scientific disciplines to natural phenomena which are understood in a different way by indigenous thinking" is called ethnoscience (Panoff & Perrin, 1973: 68). Both definitions belong to a tradition that traces back to the colonial time, when ethnography was born in the most 'developed' contries as a 'colonial science', that studied almost exclusively the cultures of subjected peoples, also as a 'science' than opposed the so-called 'primitive' thinking to the 'western' thinking as somehow absolutely different.
Among ethnographers there exists also another current, that considers the prefix 'ethno' in a very different way. E.g., Favrod characterises ethnolinguistics in his *Introduction to social and cultural anthropology* as follows: "Ethnolinguistics tries to study language in its relationship to the whole of cultural and social life" (Favrod, 1977: 90). When we transfer this characterisation of ethnolinguistics to ethnomathematics, we obtain by analogy: *"Ethnomathematics tries to study mathematics (or mathematical ideas) in its (their) relationship to the whole of cultural and social life".*
According to Crump the term ethnoscience became popular among ethnographers in the 1960's: "it may be taken to refer to the 'system of knowledge and cognition typical of a given culture'" (Crump, 1990: 160). In Crump's The *anthropology of number* (1990) there are only a few references to the work of 'ethnomathematicians'.[3] Still in the 1990's anthropologists, historians of science

[3] He points out that "first, few professional mathematicians have any interest in the cognitive assumptions in their work; second, few anthropologists are numerate in the sense of being able to realize how significant the numbers that occur in the course of their field work might be in the local culture" (Crump, 1990: viii).

and mathematicians have to find each other to develop together ethnomathematics as an anthropology of mathematics and mathematics education.[4]

Broad concept

The emergence of ethnomathematics (or ethnomathematicology) as a research field reflects the growing consciousness of the existence of many mathematics, particular in a certain way to (sub)cultures. In this sense, the following characteristics make part of the paradigm of ethnomathematics:

- Use of a broad concept of mathematics, including, in particular, counting, locating, measuring, designing, playing, explaining *(cf.* e.g Bishop, 1988);
- Emphasis and analysis of the influences of socio-cultural factors on the teaching, learning and development of mathematics;
- Mathematics (its techniques and truths) is considered a *cultural product.* Every people —every culture and every subculture— develops its own particular mathematics. Mathematics is considered to be a *universal, pan-human* activity. As a cultural product mathematics has a history. Under certain economic, social and cultural conditions, it emerged and developed in certain directions; under other conditions, it emerged and developed in other directions. In other words, the development of mathematics is *not unilinear.*

Implications for the historiography and philosopby o f mathematics

The emergence of ethnomathematics as a research field has implications for the historiography and philosophy of mathematics. Historians and philosophers cannot anymore restrict themselves to a mere reflection on the history of philosophical problems arising from 'academic' mathematics. It seems necessary to reflect about other forms of mathematical production and activity in order to understand better the relationships between nature, culture and the development of mathematical thinking.

Let me present two examples of ethnomathematical / historical studies of mathematical ideas in southern Africa.

First example: Sona geometry (cf. Gerdes, 1988, 1991, 1991a, 1991b,1993/4, 1998a, 1995, 1997, 1997a, 1999; Ascher, 1988a)

An example of a historical- ethnomathematical study is my tentative to analyse and reconstruct the sona tradition. This tradition was developed among the Chokwe of Northeastern Angola and related peoples. The Chokwe culture is well known for its decorative art that ranges from ornamentation on woven mats and baskets, iron

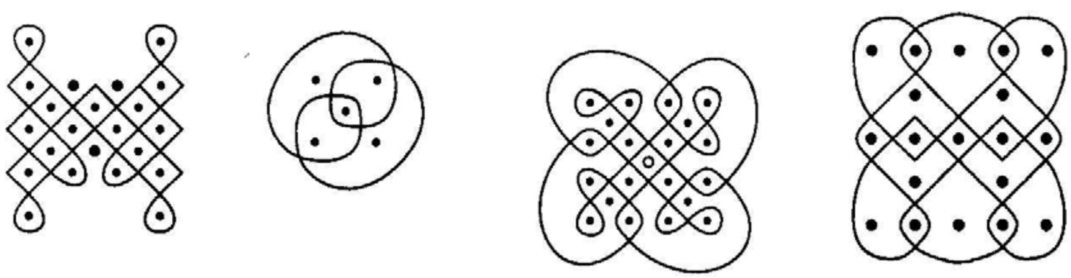
[4] D'Ambrosio uses sometimes the expression "anthropological mathematics". See e.g. D'Ambrosio (1985a), *cf.* Gerdes (1 985b).

works, ceramics, sculpture and engravings on calabash to tattooing, paintings on the walls of houses, and drawings in the sand called "sona" (singular: "lusona"). Each boy learnt the meaning and execution of the easier sona during the initiation rites. Drawing experts *(akwa kuta* sona) transmitted the more complicated sona to their male descendants. These drawing experts were at the same time the storytellers who used the sona as illustrations, referring to proverbs, fables, games, riddles, animals. The drawings were executed in the following way: After cleaning and smoothing the ground, the drawing experts first set out with their fingertips a net of equidistant points and then they draw a line figure that embraces the points of the network. The experts execute the drawings swiftly. Once drawn, the designs are generally immediately wiped out. Figure 1 presents some examples of sona.

Slave trade, colonial penetration and occupation provoked a cultural decline and the loss of a great deal of knowledge about sona. On the basis of an analysis of sona reported by missionaries, colonial administrators and ethnographers,[5] I tried to contribute to the reconstruction of mathematical elements in the sona tradition.

As the examples in figure 1 suggest, symmetry and monolinearity played an important role as cultural values: most Chokwe sona are sym-metrical and/or monolinear. Monolinear means composed of only one (smooth) line; a part of the line may cross another part of the line, but never a part of the line may touch another part.

FIGURE 1
Examples of symmetrical, monolinear *sona*

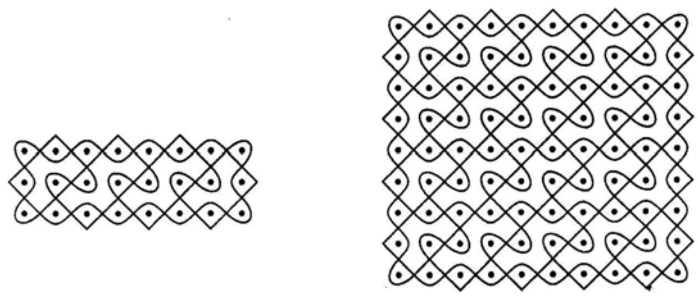


The drawing experts developed a whole series of geometric algorithms for the construction of monolinear, symmetrical designs. Figure 2 displays two monolinear sona belonging to the same class in the sense that, although the dimensions of the underlying grids are different, both sona are drawn applying the same geometric algorithm.

[5] The most important collections of sona were published by: Hamelberger, 1952; Santos, 1961; Pearson, 1977; Fontinha, 1983, Kubik, 1988.

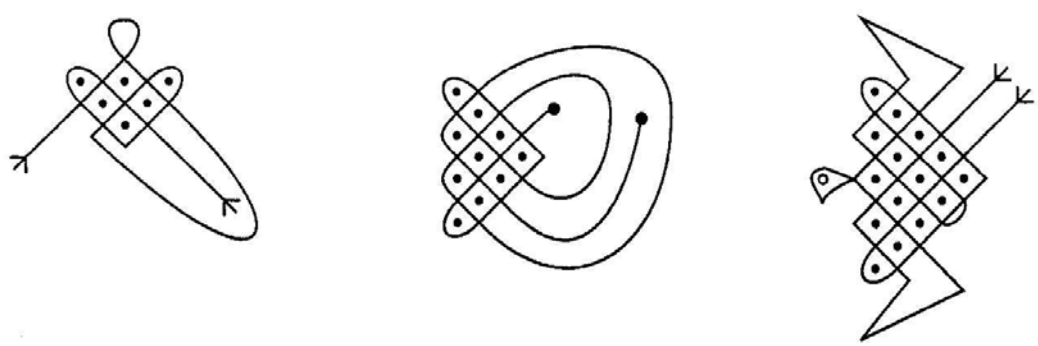
FIGURE 2

Two sona drawn with the same geometric *algorithm*



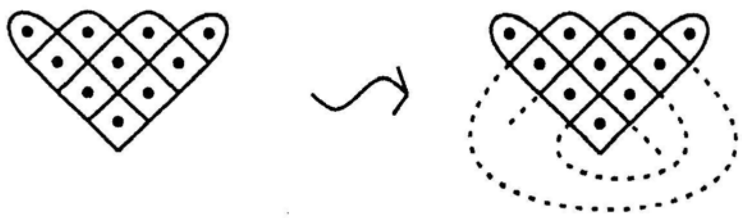
The drawing experts also invented various rules for building up monolinear sona. The following presents a first example. Figure 3 shows three, monolinear sona. They are similar to each other: each presents a basic design of triangular form. I suppose that the drawing experts who invented these sona probably began with triangular patterns and transformed them into monolinear patterns with the help of one or more loops (see the example in figure 4). The monolinear patterns so obtained were adapted (maybe later by others) topologically so that they could express the ideas the drawers wanted o transmit through them.

FIGURE 3



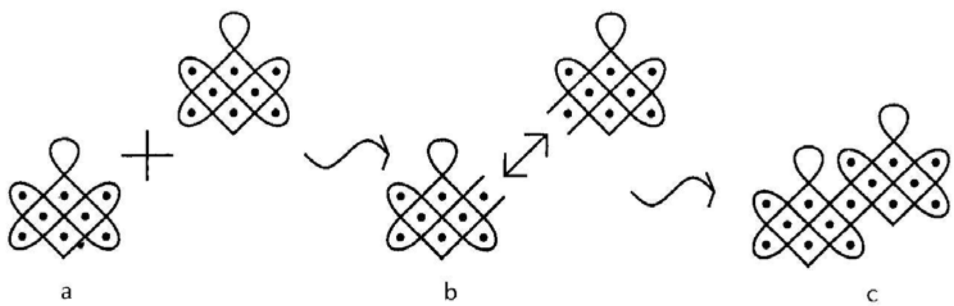
An eagle carrying a chicken a | A person dead to fatigue b | A *thimunga* bird in flight c

FIGURE 4
Transformation of triangular design into a monolinear design



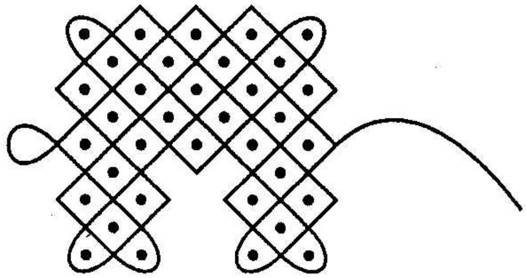
Sona experts also discovered various rules for chaining monolinear sona to form bigger monolinear sona. Figure 5 displays an example of the use of a chain rule: it indicates how the appearance of the monolinear drawing in figure 5c may be explained on the basis of the monolinearity of the two patterns in figure 5a and the way they have been chained together (see Figure 5b).

FIGURE 5
Examples of the application of a chain rule

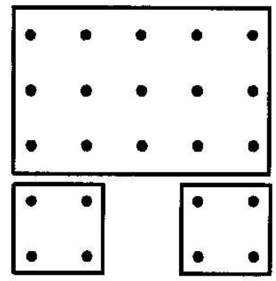


The monolinearity of the lusona in figure 6a may be explained on the basis of another chain rule: If one joins square grids (in the example: two grids of dimensions 2 x 2) to a rectangle with dimensions which are relative prime (in the example: 3 x 5), then the resulting grid leads to a monolinear drawing if one applies the same geometric algorithm as used in figure 6c with the rectangular grid of dimensions 3x5 ('diagonally plaited mat' algorithm).

FIGURE 6



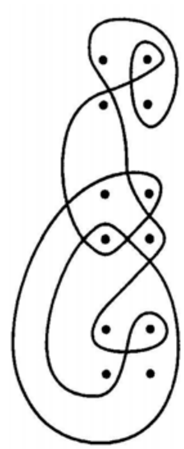
Representation a monkey-dog
a



Grid structure
b

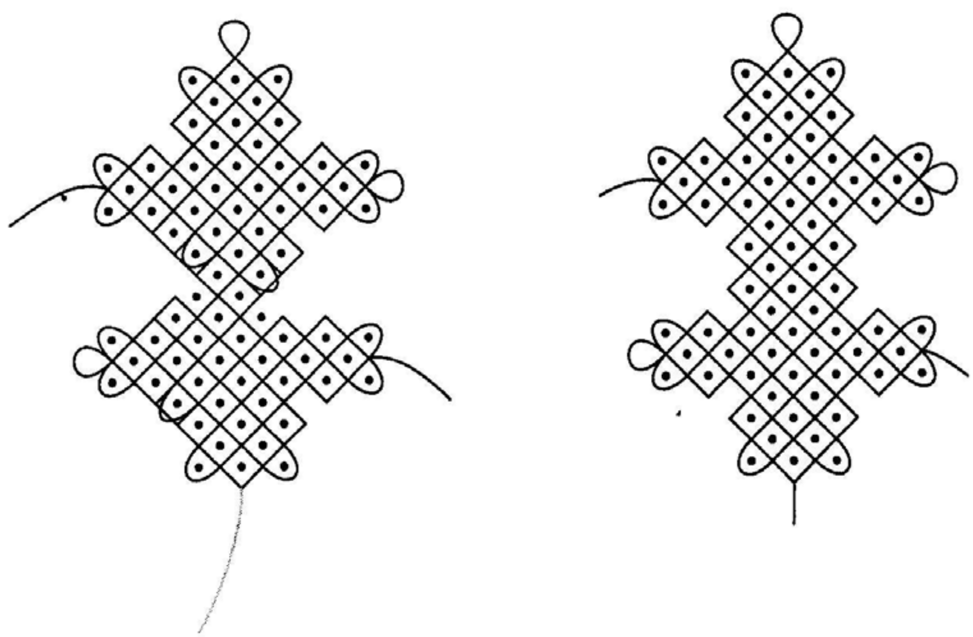
When analysing and reconstructing elements of the sona tradition, I found that there are several reported sona, which clearly do not conform to the cultural values of symmetry and monolinearity. Sometimes the symmetry or monolinearity was broken in order to give the drawing a specific meaning (Figure 7 gives an example).

FIGURE 7
Lusona representing ‘kilu' or 'kalamba', the thinker
(monolinear, but not symmetrical)



More often we seem to be dealing with mistakes or errors. Figure 8a gives an example of a reported lusona with mistakes and figure 8b the reconstructed drawing without mistakes. The drawing experts may have committed some of these mistakes, because they were contacted when they were already old men. They were saying that as young men they had been much better 'akwa kuta sona'. We may be dealing with errors in the transmission of the sona knowledge from one generation to the next, or with mistakes on the part of the reporter, who had little time to make his copies, as traditionally the drawing experts wipe their drawings out immediately after concluding their story. The wiping out was one way to protect their knowledge, to maintain their monopoly of sona knowledge. Here we have another probable reason for mistakes in the reporting of sona: a form of cultural resistance; the experts made consciously mistakes to deceive the reporter —the white man, associated with slave trade, colonial administration and Christianity—, and so to protect their secret knowledge. In the secret and monopoly character of the sona tradition lies also a reason for its gradual extinction: as soon as a drawing expert is taken slave, the knowledge disappears from his community; some of the knowledge may have survived in one way or another in the 'New World'.[6] In 1997, the author had the opportunity to learn about the survival of a sona type drawing in the United States of America. Figure 9 presents a sand drawing I received from a woman who grew up in Greenwood, Mississippi which is in the Delta near Greenville. She had learned this and other sona from Mary Reaves —of African descendance, born almost a hundred ago— who was her nurse (Susan Enger, personal communication, 1997).

FIGURE 8

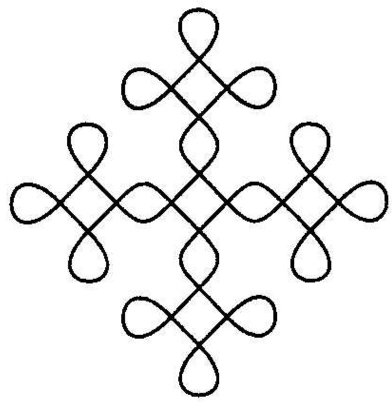


Reported lusona, representing a lioness with her two cubs. The drawing is neither symmetrical nor monolinear
a

Reconstructed symmetrical and monolinear line drawing (the tails are added at the end)
b

[6] *Cf.* the exceptional arithmetical knowledge of Thomas Fuller, who was brought in 1 724 as a slave from África to North America. See Fauvel & Gerdes (1990).

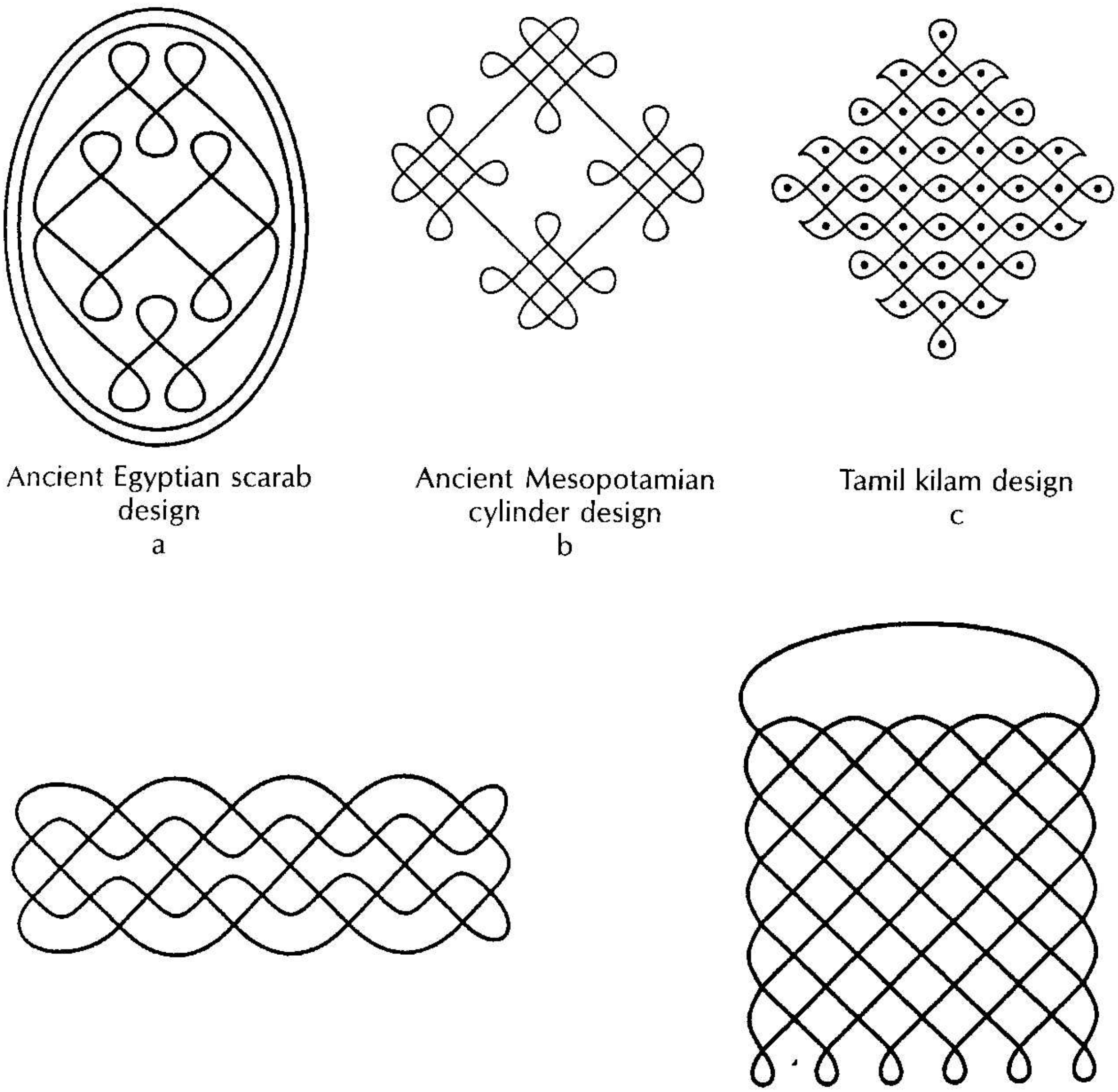
FIGURE 9
A lusona type drawing —that survived the slave trade— in the USA



The same line drawing also appears among the Tamil threshold designs in India and on decorated cloths from Sierra Leone and Ethiopia. Cf. P. Gerdes, "Ethnomathematik dargestellt...", o.c., pp. 374, 396, 397.

The historical reconstruction of mathematical ideas incorporated in the sona tradition led me not only to mathematical and didactic re-search, but also to the study of traditions that from a mathematical-technical point of view display similarities with the sona tradition. So far I analysed some geometrical algorithms in Ancient Egypt (used for the decoration of scarabs, vases and walls), monolinear designs from Ancient Mesopotamia, threshold designs drawn by Tamil women in India (cf. Gerdes, 1989, 1995a), Celtic knot designs (cf. Cromwell, 1993; Gerdes, 1999b), sand drawings from the Vanuatu islands (Oceania) (cf. Ascher, 1988), and some designs motifs from other parts of Africa. I suppose that the methodology used in the reconstruction of the sona tradition may be further developed in the study of these other traditions that bear certain similarities with the sona tradition. Figure 10 presents examples of symmetrical, monolinear designs belonging to these traditions.
Concerning the historiography of mathematics, the example of the sona raises the question of what type of sources and argumentation can be used. Also it shows —in the case of Ancient Egypt— that various traditions with mathematical 'ingredients' may co-exist in the same society, interrelated with each other or not. In that sense, it becomes difficult, if not impossible, to speak of the mathematics of Ancient Egypt, the mathematics of Greece, etc.

FIGURE 10
Examples of symmetrical and monolinear designs



Ancient Egyptian scarab design
a

Ancient Mesopotamian cylinder design
b

Tamil kilam design
c

Celtic knot design
d

Sand drawing design from Vanuatu
e

Second example: Litema geometry
(cf. Gerdes, 1995b, 1996a, 1998b: ch. 9)

In Lesotho and neighbouring zones of South Africa, Sotho women developed a tradition of decorating the walls of their houses with designs. The walls are first neatly plastered with a mixture of mud and dung, and often colored with natural

dyes. While the last coat of mud is still wet, the women engrave the walls, using their forefinger. Their art is seasonal: The sun dries it and cracks it, and the rains wash it away. An entire village is redecorated before special occasions such as engagement parties, weddings, and important religious celebrations.
The Sotho women call their geometric patterns, litema (singular: tema). The books The African Mural (Changuion et al., 1989) and African Painted Houses: Basotho Dwellings of Southern Africa (Wyk, 1998) contain beautiful collections of photographs of litema. The National Teacher Training College of Lesotho published a colection litema patterns, collected by its mathematics students (Mothibe, 1976). In his presentation, the coordinator underlines that "like other national traditions this one is in danger of dying out as more and more houses are built of concrete walls which are usually painted or white-washed. Also a growing number of women no longer like or know the art anymore" (Mothibe, 1976:2).
Symmetry is a basic feature of the litema patterns. Figure 11 presents part of a tema pattern. As ¡s often the case, this tema pattern is built up from a basic square that constitutes the (unit) cell of the pattern. Figure 12 displays the cell for the tema in figure 11.

FIGURE 11

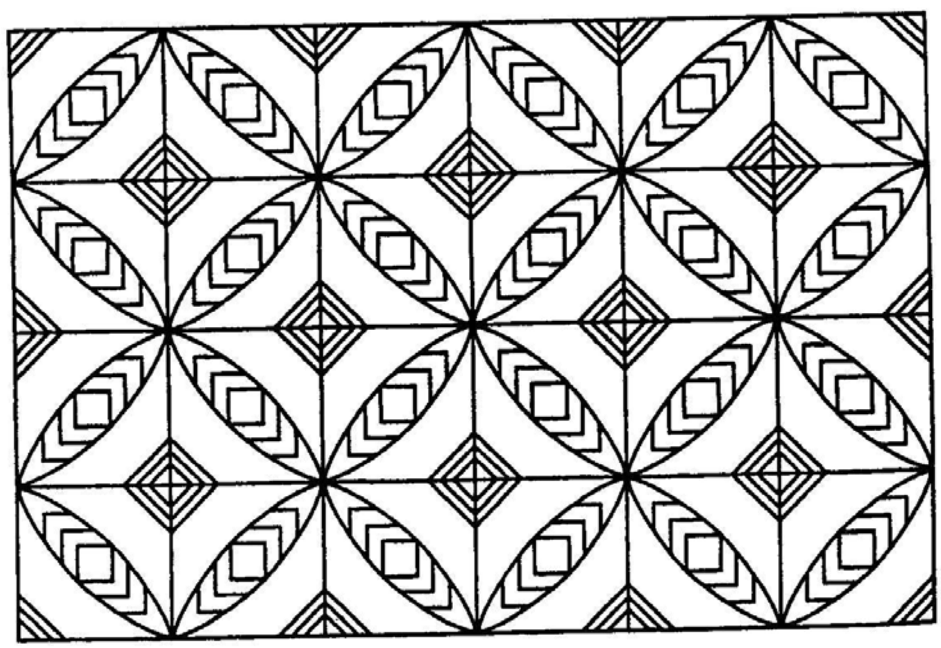
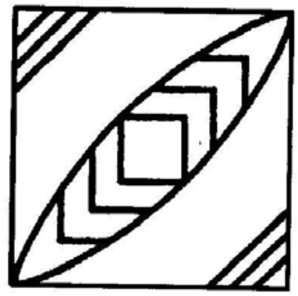


FIGURE 12



The Sotho women lay out a network of squares and then they reproduce the basic design in each square. The number of reproductions or repetitions of the unit cells depends, in practice, on the available space on the wall to be decorated. As in figure 11, a whole pattern is built up out of repetitions of a 2 x 2 square, in which the unit cell appears ¡n four positions, obtained by horizontal and vertical reflection about the axes of the 2 x 2 square. The symmetries of a whole pattern depend on the symmetries of the unit cell. The unit cell in figure 12 has two diagonal axes of symmetry. The unit cell of the litema pattern in figure 13 has no axial symmetry however is invariant under a half turn. The unit cell of the litema pattern in figure 14 has one axis of symmetry.

FIGURE 13

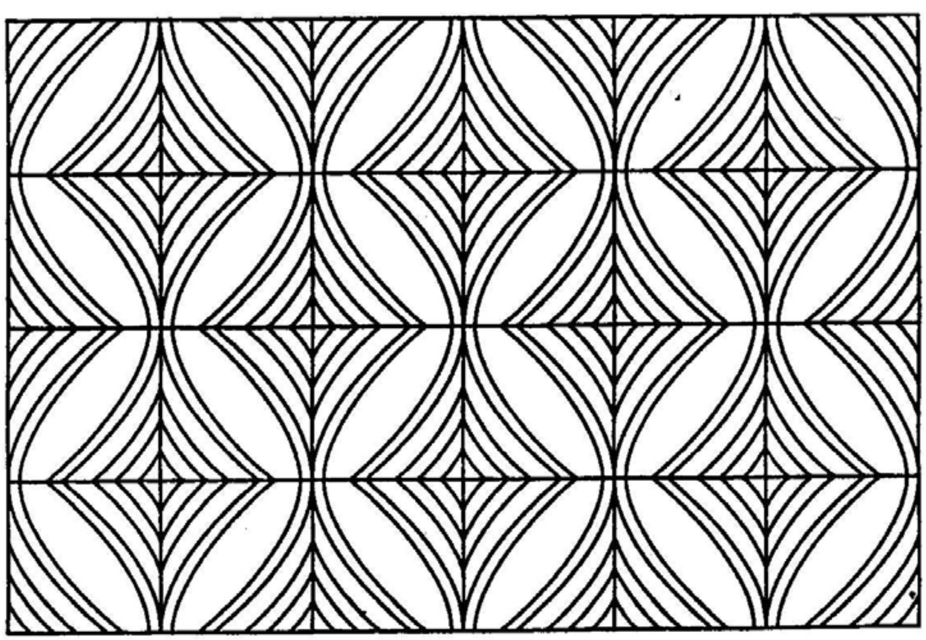
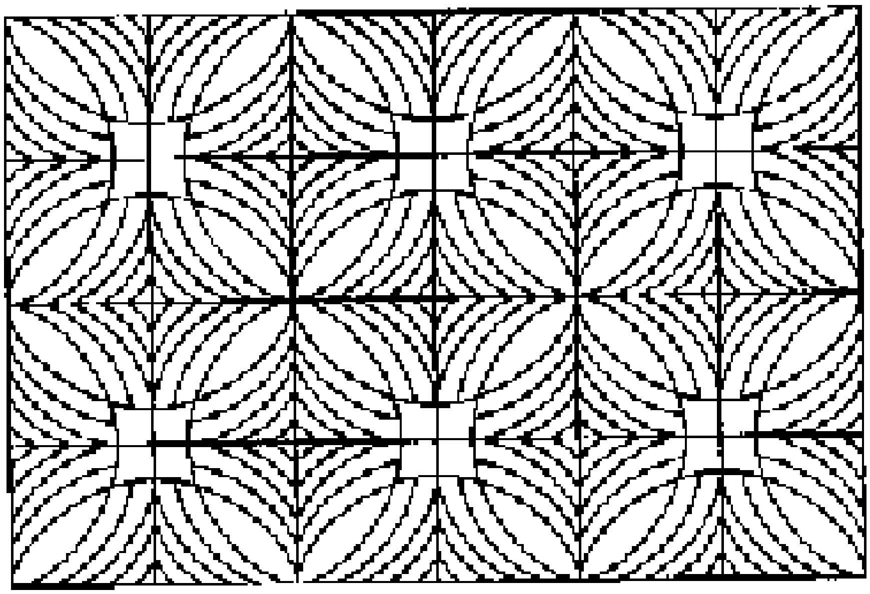
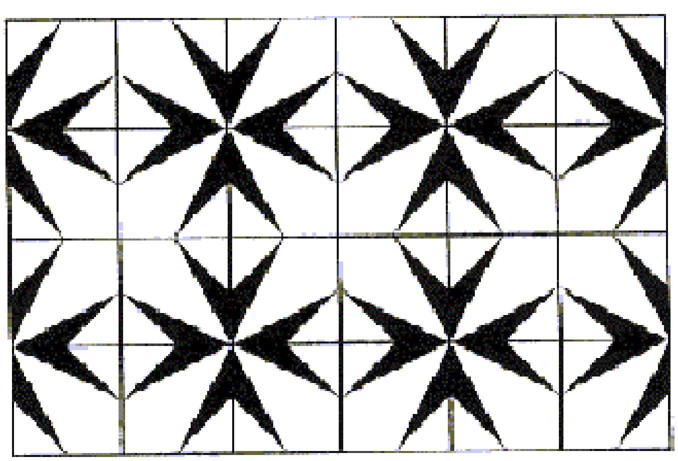


FIGURE 14



Several painted litema, and others whereby changes in the relief of the dung surface of the plater suggest two distint colors, may be represented on paper as black-and-white patterns. Some, as the one in the figure 15, are built up in the same way as the earlier patterns considered.

FIGURE 15



Others are two-color patterns, whereby in each horizontal or vertical reflection of the unit cell the colors are reversed. The image of a unit cell is the negative (in photographic terms) of the reflected cell (see the example in figure 16, leading to the tema pattern in figure 17). Figure 18 present further examples.

FIGURE 16

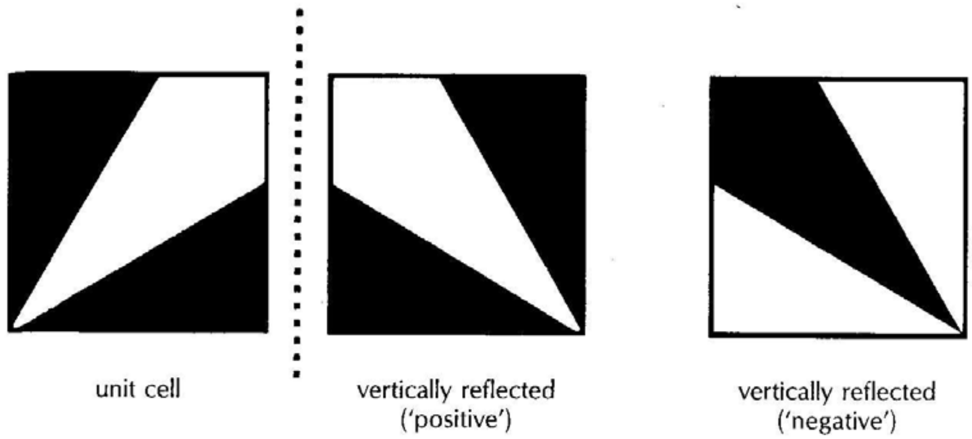


FIGURE 17

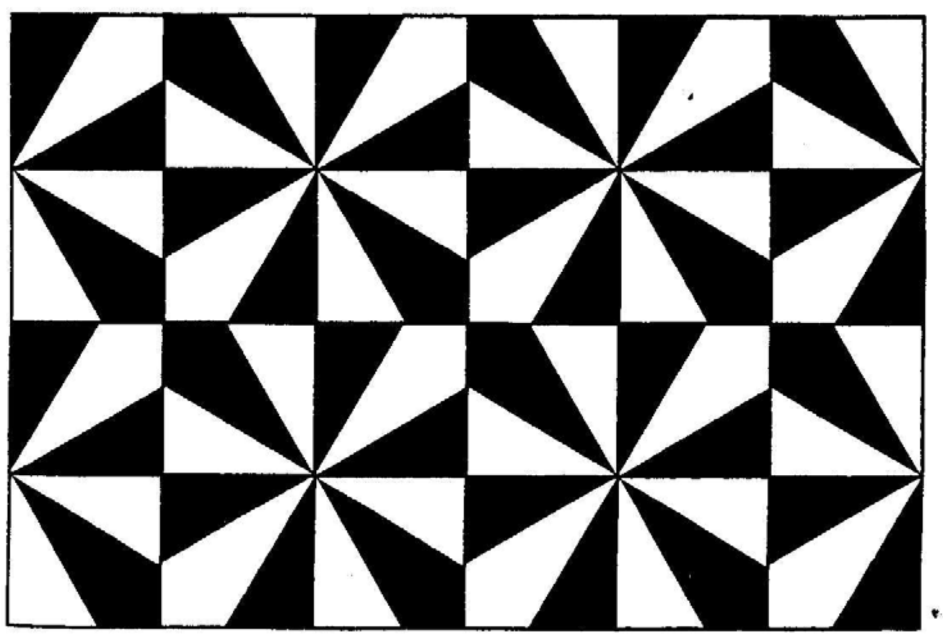
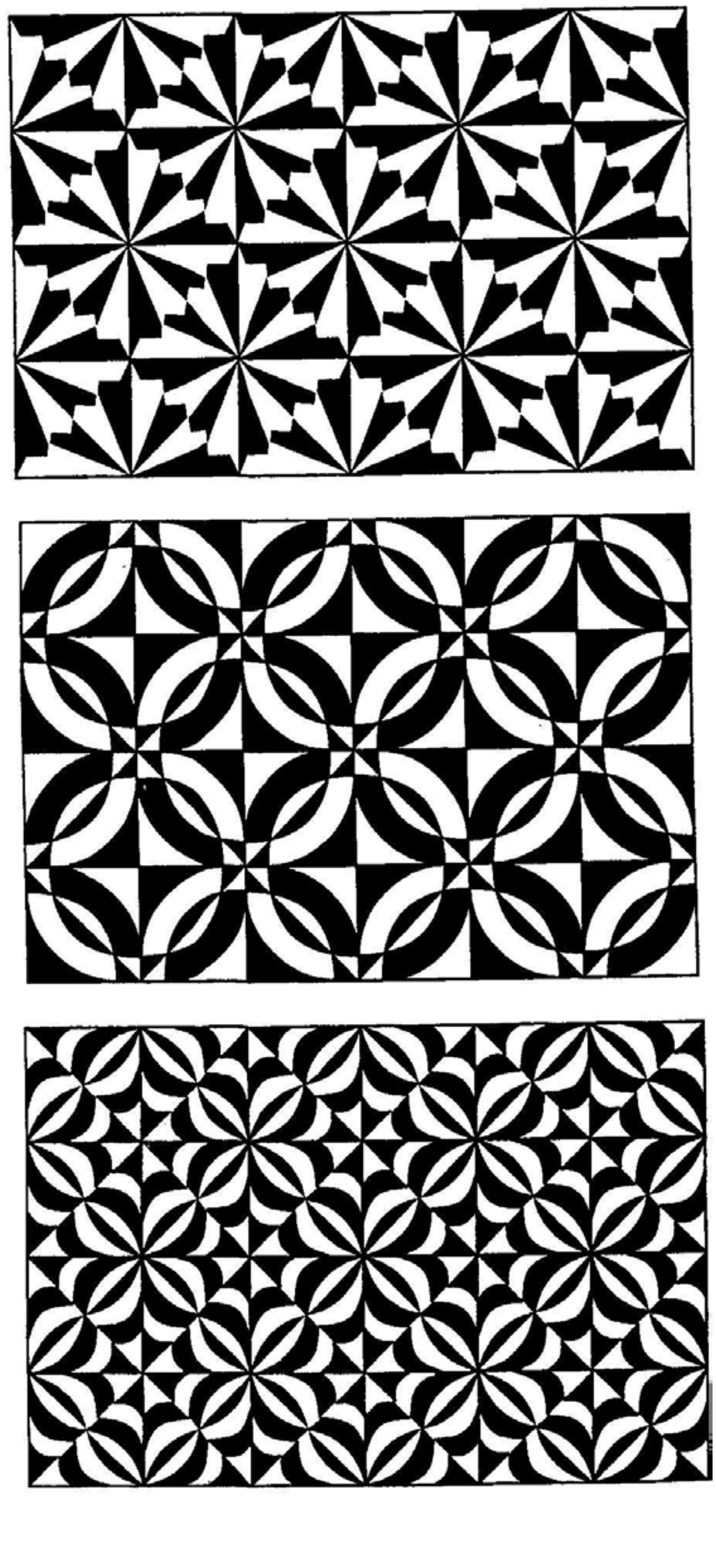


FIGURE 18



A different, and colorful style of geometric wall decoration has been developed by Ndebele women in south Africa (cf. Courtney-Clarke, 1986; Powell & Lewis, 1995; Gerdes, 1995b, 1996a, 1998b, ch. 10). Suggestions for further research of female cultural activities in Southern Africa in which mathematical considerations are embedded, are presented in Women, Art and Geometry in Southern Africa (Gerdes, 1995b, 1996a, 1998b; cf. Gerdes & Bulafo, 1994).
One question for the historiography of mathematics, raised by this example of ethnomathematical research, is the underrepresentation of women in the history of mathematics. Does this underrepresentation correspond to reality, or does it reflect certain views about mathematics? Does it reflect a view of mathematics as 'something' that can be 'easily' dissected from other cultural elements? Ethnomathematical research sems to show that mathematical ideas and activities are often 'interwoven' with other cultural ideas and activities. Underrepresentation of any cultural group in the history of mathematics, may tell us more about the historiography of mathematics than about the underrepresented cultural groups. Here lies also a challenge for the philosophy of mathematics: whose mathematics is the object of philosophical analysis?

Mathematical thinking and Ethnomathematics

Conflicting views

The question may be raised if the examples given are an expression of mathematical thinking? If one reads the ethnographer Santos on the mathematical knowledge of the Chokwe, his answer would be no: The Chokwe know some arithmetic, some time reckoning and some geo-metrical vocabulary (line, curve, point,...), but they do not know mathematics (Santos, 1960). The same author published, however, an interesting study on the sona (Santos, 1961), but he did not see any relationship with mathematics. His 'no' answer reflects the horizon of his ethnographic training and his school mathematics education in the colonial metropole. But does this school mathematics education give a real picture of what mathematics is all about? Are the concepts, are the construction and chaining rules, ...developed and invented by the 'akwa kuta sona' mathematical? Ever since my first personal contact with the sona, I 'felt' —trained in Europe as a research mathematician— that I was dealing with mathematical ideas, and speaking at professional gatherings of mathematicians I saw that the mathematical aspects and potential of the sona was quinckly absorbed by the mathematical community. This example may serve to raise the important question of who defines some activity, some idea, or some theory as mathematical. Who defines "What is m mathematical thinking?" What can be said about his/her socio-cultural background? Is it possible to discuss philosophical questions related to mathematics platonically outside the con-text of culture?

Intercultural intelligibility o f mathematical thinking

As ethnomathematics (or ethnomathematicology) is the field of inquiry that studies mathematical ideas in their historical-cultural contexts, an ethnomathematician may be, for instance, interested in understanding the role and embodiment of mathematical thinking both in the invention and (re)production of decorative patterns. If no direct dialogue, physically or historically, with the inventors and/or (re)producers is possible, the researcher may try to reconstruct elements of mathematical thrinking probably involved in the invention and reproduction processes. These reconstructed elements may be called 'frozen' or 'hidden' mathematics (cf. Gerdes, 1990, 2000). The reconstruction is not completely impossible as the researcher may have developed some feeling for mathematical ideas. Just like any musician (or even any human being) may develop a certain understanding of and feeling for musical expressions, and any linguist (or even any human being) may develop a certain understanding and recognition of language phenomena. In this sense, mathematical thrinking is as pan-human as using a language or involvement in music (playing, listening,...). This 'feeling for' is the result of an enculturation process (cf. Bishop, 1988).
From an ethnomathematical perspective, mathematics becomes the product of all cultures, being the school mathematics experience of a researcher only one form of mathematical experience. Mathematics is not the product of a particular culture sphere, 'western', but a common human experience. In the process of studying mathematical ideas in diverse cultural contexts, the understanding of what mathematics is, or better of what constitutes mathematical activity, may be deepened. Mathematical thinking is only interculturally intelligible. And there lies a challenge for philosophy in general and for the philosophy of mathematics in particular: to contribute to the development of the means for intercultural intelligibility.

Responsibility and urgency

In other words, ethnomathematical studies may broaden the (intercultural) understanding of what are mathematics, of what are mathematical ideas and activities. There cannot be a sole, unified view of mathematics. For a monolinthic and dominant view there is no basis. At the same time, for the other extreme, a cultural relativism concerning mathematics, there is also no ground: intercultural intelligibility seems possible.
Ethnomathematicians, historians and phylosophers of mathematics may arrive at these opinions, but, in this phase of 'globalisation' monolithic views seem still to be dominant among educators, professional mathematicians and administrators. This raises the question of the responsibility of ethnomathematicians, historians and philosophers of mathematics. Relative to research, it is my opinion that among the most urgent tasks is the study of mathematical elements in cultural spheres that are under siege, in particular of cultures in the "Third World", in the South. Humanity will loose an enormous source of knowledge, an enormous potential for

broadening the reflection on mathematical thinking if these elements are not studied today and in the near future.

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